Christ Church Grammar School

MATHEMATICS METHODS Year 11
Section One:
Calculator-free

Your name _____ SOLUTIONS & MARKING KEY _____

Teacher's name _____

Time and marks available for this section
Reading time for this section: 3 minutes
Working time for this section: 15 minutes
Marks available: 17 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
                           correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1.   The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer Booklet.

3.   Answer all questions.

4.   You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5.   Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6.   **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7.   It is recommended that **you do not use pencil**, except in diagrams.

Question 1 (2 marks)

In Australia, the probability of having blue eyes is approximately 35% and the probability of having fair hair is approximately 24%. The probability of having both blue eyes and fair hair is approximately 17%.

Determine the probability that someone with blue eyes will have fair hair.

$$P\left(\text{fair hair} \mid \text{blue eyes}\right) = \frac{17}{35}$$
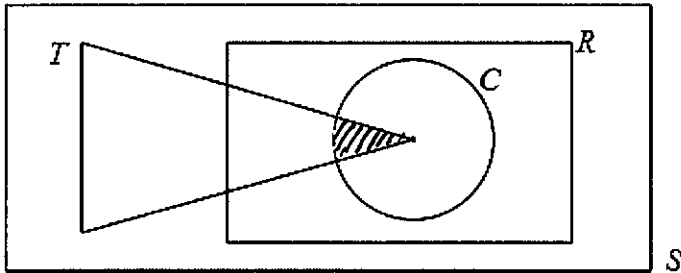
✓ correct numerator

✓ correct denominator

Question 2 (9 marks)

Sets T, R and C are defined in a sample space S. No region within the sample space S is empty.

(a)   Shade the appropriate region to represent the set $T \cap R \cap C$.   (1 mark)
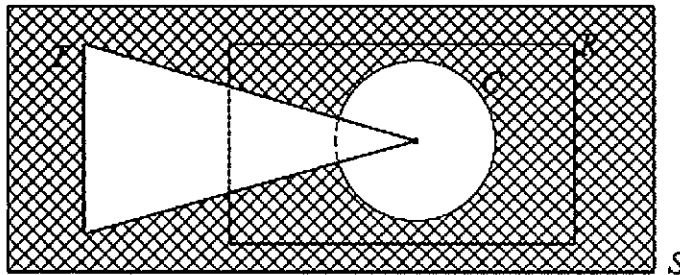

✓ correct region shaded

(b)   Use set notation to represent the shaded region in the following diagrams:
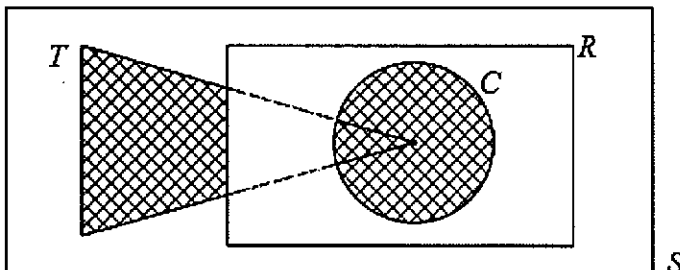
(i)

(2 marks)

$\overline{T \cup C}$ or $\overline{T} \cap \overline{C}$

✓ writes union of T and C

✓ writes the complement of this union

(ii)

(2 marks)

$C \cup (T \cap \overline{R})$

✓ writes the union with C

✓ writes the intersection of T with the complement of R

See next page

Question 2 continued

(c) State whether the following statements are true or false. Justify your answer.

(i) R and $\overline{R}$ are mutually exclusive. (2 marks)

True.

Since $R \cap \overline{R} = \emptyset$

or the sets R and $\overline{R}$ do not overlap in the Venn diagram.

✓ states the statement is true

✓ justifies their answer

(ii) R and C are independent events. (2 marks)

$P(R|C) = 1$ since C is a subset of R

Also, $P(R) < 1$ (from Venn diagram)

∴ $P(R|C) \neq P(R)$

Hence R and C are NOT independent events.

✓ states the statement is false

✓ justifies their answer

See next page

Question 3   (6 marks)

The 6th, 7th and 8th rows of Pascal's triangle are shown below. This information may be used to answer the questions that follow.

Row 6		1	6	15	20	15	6	1	
Row 7	1	7	21	35	35	21	7	1	
Row 8	1	8	28	56	70	56	28	8	1

(a)   Evaluate $\binom{8}{2}$. $= \underline{\underline{28}}$ (1 mark)

                               ✓ states correct value

(b)   Evaluate $\binom{9}{4}$. $= 56 + 70$ (2 marks)

             $= \underline{\underline{126}}$

                       ✓ uses value in 8th row to correctly
                         determine the value in the 9th row

                       ✓ states correct value

(c)   If $(1-2x)^6 = a + bx + cx^2 + \ldots$ determine the value for a, b and c.   (3 marks)

$$(1-2x)^6 = 1^6 + 6(1)^5(-2x) + 15(-2x)^2 + \ldots$$

$$= \underline{\underline{1 - 12x + 60x^2 + \ldots}}$$

                              ✓ correct value for a
                              ✓ correct value for b
                              ✓ correct value for c

End of Questions

Christ Church Grammar School

MATHEMATICS METHODS Year 11
Section Two:
Calculator-assumed

Your name _____ SOLUTIONS & MARKING KEY _____

Teacher's name _____

Time and marks available for this section
Reading time for this section: 4 minutes
Working time for this section: 25 minutes
Marks available: 26 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates and up to three calculators approved for use in the ATAR examinations

Important note to candidates
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Question 4 **(5 marks)**
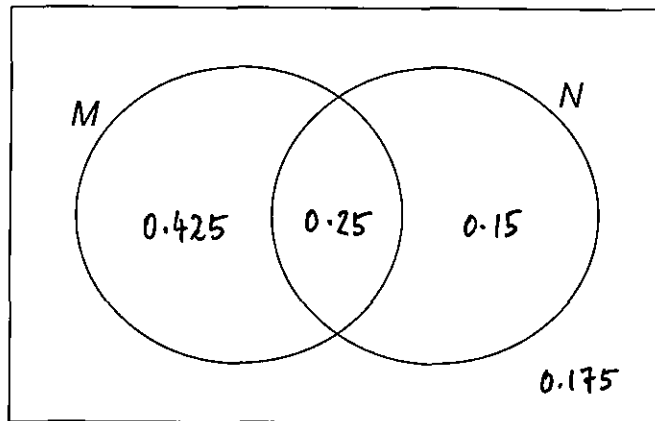
Two events M and N are such that:

$P(M) = 0.675$　　　　　$P(M \cap N) = 0.25$　　　　$P(\overline{M} \cap N) = 0.15$

(a)　Complete the Venn diagram below.　　　　　　　　　　(2 marks)

✓ writes 0.425 and 0.25
in the correct spaces

✓ writes 0.15 and 0.175
in the correct spaces

(b)　Determine $P(M \cup N)$　　　　　　　　　　　　　　(1 mark)

$$P(M \cup N) = 0.675 + 0.15$$
$$= \underline{\underline{0.825}}$$

✓ correct answer

(c)　Determine $P(N|M)$　　　　　　　　　　　　　　(2 marks)

$$P(N|M) = \frac{0.25}{0.675}$$

$$= \underline{\underline{\frac{10}{27}}} \quad or \quad 0.37 \ (2d.p)$$

✓ correct numerator (10)

✓ correct denominator (27)

Question 5 (11 marks)

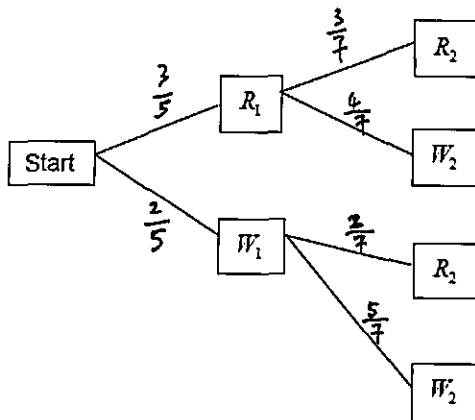An experiment is conducted using two separate bags, Bag 1 and Bag 2.

Bag 1 has 3 red and 2 white balls.
Bag 2 has 2 red and 4 white balls.

A ball is randomly selected from Bag 1 and is then placed into Bag 2.
Then a ball is randomly selected from Bag 2.

A tree diagram can be drawn showing the possibilities:

Note: R_1 is the event a red ball was selected from Bag 1.

R_2 is the event a red ball was selected from Bag 2.

(a) Explain why events R_1 and W_1 are considered to be both mutually exclusive and complementary events. (2 marks)

They are mutually exclusive since $P(R_1 \cap W_1) = 0$ ie., both cannot occur simultaneously.

They are complementary since $P(R_1 \cup W_1) = 1$ or $P(R_1) = 1 - P(W_1)$

✓states why R_1 and W_1 are mutually exclusive
✓states why R_1 and W_1 are complementary

(b) Determine $P(R_1)$ (1 mark)

$P(R_1) = \frac{3}{5}$ or 0.6

✓correct probability

See next page

Question 5 continued

(c) Determine the following probabilities:

(i) $P(W_2 \mid R_1)$ $= \dfrac{4}{7}$ (2 marks)

✓ correct numerator
✓ correct denominator

(ii) $P(R_1 \cap W_2)$ (2 marks)

$= P(R_1) \times P(W_2 \mid R_1)$

$= \dfrac{3}{5} \times \dfrac{4}{7} = \dfrac{12}{35}$ or $0.34\,(2dp)$

✓ forms a product of probabilities from (b) and (c)(i).
✓ correct probability

(iii) $P(W_2)$ (2 marks)

$= \dfrac{3}{5} \times \dfrac{4}{7} + \dfrac{2}{5} \times \dfrac{5}{7}$

$= \dfrac{22}{35}$ or 0.6285

✓ forms a sum of probabilities considering the possible 1st bag outcome

✓ correct probability

(d) Determine the probability that a white ball was selected from Bag 1, given that a white ball is selected from Bag 2. (2 marks)
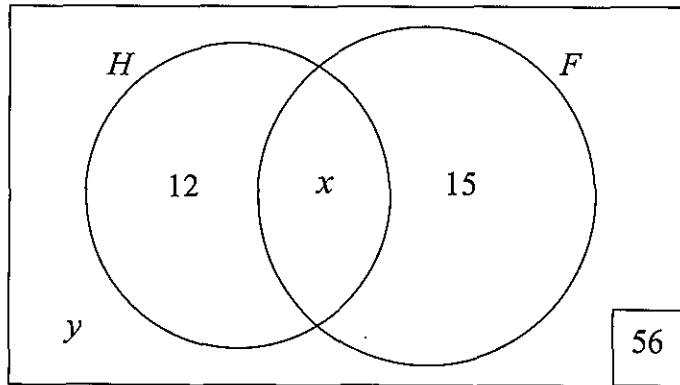
$P(W_1 \mid W_2) = \dfrac{\frac{2}{5} \times \frac{5}{7}}{\frac{22}{35}}$

$= \dfrac{5}{11}$ or 0.4545

✓ forms correct conditional probability
✓ correct probability

See next page

Question 6 (8 marks)

For a particular group of 56 Year 11 students, the Venn diagram shown below indicates the number of students who play either hockey, football or both.

(a) The diagram indicates that $n\left(\overline{H \cup F}\right) = y$.

Describe in words what this means in this specific context. (1 mark)

There are y students who do not play either hockey or football.

✓ states correct statement

(b) A student is selected at random from the group of 56 students.
Write an expression for the **probability** that this student plays hockey or football.

(2 marks)

$$P(H \cup F) = \frac{12 + x + 15}{56}$$

$$= \frac{x + 27}{56}$$

OR $P(H \cup F) = 1 - \frac{y}{56}$

i.e., $\frac{56 - y}{56}$

✓ applies the probability of the union correctly
✓ writes a correct expression in terms of either x or y.

See next page

Question 6 continued

(c)   A student is selected at random from the group of 56 students.
Write an expression for the **probability** that this student plays hockey given that they
play football.

(2 marks)

$$P(H|F) = \frac{x}{x+15}$$

✓ applies the probability of the conditional event
H given F.

✓ writes correct expression in terms of x.

(d)   If playing hockey and playing football are independent events, determine the possible
value(s) of x.
(3 marks)

If H and F are independent events, then $P(H|F) = P(H)$

$$\therefore \quad \frac{x}{x+15} = \frac{x+12}{56}$$

CAS SOLVE : $x = 9$ or $x = 20$

Ⓞ️Ⓡ️ If H and F are independent events , then $P(H \cap F) = P(H) \times P(F)$

$$\therefore \quad \frac{x}{56} = \frac{x+12}{56} \times \frac{x+15}{56}$$

CAS SOLVE : $x = 9$ or $x = 20$

✓ states the condition for independent events
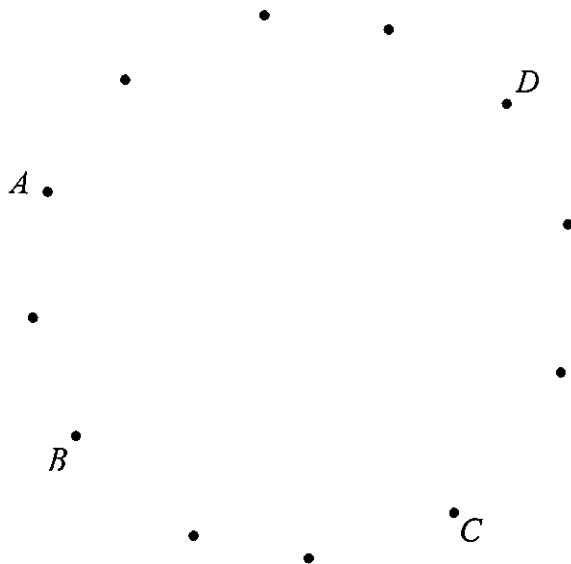correctly in terms of H and F

✓ forms correct expressions in terms of x

✓ solves correctly to give both values of x.

Question 7   **(2 marks)**

The diagram below shows 12 points where no three points are collinear. Points A, B, C and D define a quadrilateral.

Quadrilateral $ABCD$ is one of the many quadrilaterals that can be formed.
Determine the number of quadrilaterals that can be formed.

Number of quadrilaterals $= \ ^{12}C_4$

$= \underline{\underline{495}}$

✓ states that a combination of 4 points
is required from 12

✓ correctly evaluates the combination coefficient.

End of Questions